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Investigation the Nuclear Binding Energy per nucleon of selected Nuclei

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ABSTRACT

Objective: *In the current study nuclear binding energies per nucleon E_b of a selected group of nuclei* $\left({}^{3}_{1}H,\ {}^{26}_{12}Mg,\ {}^{34}_{16}S,\ {}^{50}_{23}V,\ {}^{56}_{26}Fe,\ {}^{61}_{28}Ni,\ {}^{80}_{36}Kr,\ {}^{116}_{50}Sn,\ {}^{204}_{80}Hg\ and\ {}^{238}_{92}U\right)$ *has been computed, using two equations namely the principle equation and the Weizsäcker empirical equations, and compared their results with experimental values of the E_b of these nuclei.* ***Method:*** *The study calculated the nuclear binding energies per nucleon of selected nuclei using the principle equation and the Weizsäcker empirical equation, and then evaluated their performance by comparing the calculated values with experimental binding energy data.* ***Results:*** *This investigation concluded that the theoretical calculation using the Weizsäcker formula is more appropriate than the principle equation, except for the nucleus of the hydrogen isotope (Tritium), where the deviation was very large due to the equation's dependence on the atomic and mass numbers and multiple correction factors, which caused a large deviation. The research also concluded that light nuclei often tend to fuse due to their spread in the fusion region, while nuclei with a mass number close to 60 show high stability, such as iron and nickel, because their binding energies per nucleon are large compared to other nuclei. Nuclei with large atomic numbers tend to fission because this increases the Coulomb term effect, which reduces the nuclear binding energy per nucleon.* ***Novelty:*** *The novelty of this study lies in the comparative evaluation of the principle equation and the Weizsäcker empirical equation across selected light, medium, and heavy nuclei, providing a comprehensive assessment of their accuracy in predicting nuclear binding energies per nucleon and relating the results to nuclear stability, fusion, and fission behavior.*

INTRODUCTION

A nucleus model is a theoretical framework used in nuclear physics to describe the structure and behavior of atomic nuclei. These applied models analogies and mathematical descriptions to explain the properties and interactions of protons and neutrons within the nucleus. Nuclear models are essentially theoretical constructs that help us understand the complex world inside the atom's nucleus [1]. Nuclei are made up of protons and neutrons (nucleons), and they exhibit a variety of behaviors. Models are needed to explain how these particles interacts, how nuclei are formed, and why some are stable while others are not. As well as One of the most important goals in nuclear physics is that explanation the properties of nuclides such as density, mass, volume, binding energy, symmetry, and others [2]. The nuclear binding energy represents the difference between the sum of the masses of the individual components of the nucleus and the nuclear mass $M_N(A,Z)$. This is called the mass defect (Δm) or mass deficiency.

The energy equivalent to this mass is called the total binding energy of the nucleus, symbolized by $BE(A,Z)$. The total binding energy depends directly on both the atomic number (Z) and the mass number (A), meaning that its value varies from one nucleus to another. It has been found that the nuclear binding energy is less variable than the total binding energy. The total binding energy can be calculated practically from accurate mass calculations using mass spectrometry or from separation energy calculations [3-4].

It has been proven that the size of a nuclide is directly proportional to its mass number, and this proportionality also applies to liquids. A nuclear model proposed by the German physicist Carl Friedrich von Weizsäcker in 1935 was used. In this model, the atomic nucleus was described as analogous to a liquid molecule. The reasons for this analogy are several, including the constancy of nuclear density and its independence from volume; the binding energy of nucleons to each other is equivalent to the surface tension of liquid molecules; the evaporation of a liquid droplet is equivalent to the radioactive decay properties of nuclei; the internal thermal vibrations of the molecule are equivalent to the phenomenon of a complex nucleus; and nuclides are similar to incompressible liquid drops of enormous density. Since the nuclear force is saturated and the average binding energy per nucleon is constant, Weizsäcker was able to present his model [5-6].

In the liquid droplet model, nucleons are not described individually, but are considered average values. Therefore, this model successfully described some properties of nuclei, such as the average binding energy per nucleon, while it did not provide many other nuclear properties, such as excited nuclear states, magic numbers, and nuclear magnetic moments [7].

RESEARCH METHOD

First Method: The Primary Equation

Nuclear binding energy $BE(A,Z)$ is defined as the work required to break a nucleus apart into its components, or the energy released when individual nuclei are brought together. The primary nuclear binding energy is calculated from the following equation [8-9]:

$$BE(A,Z)_{Primary} = [\, Zm_p + Nm_n - M_N(A,Z)]c^2 \qquad (1)$$

The values of the constants $(m_p,, m_n, c^2)$ are $\left(1.007276 \text{ amu}, 1.008665 \text{ amu}, 931.5 \frac{\text{MeV}}{\text{amu}}\right)$ respectively.

The average nuclear binding energy per nucleon can be calculated from the following formula [8-9]:

$$BE_{avg} = \frac{BE(A,Z)}{A} \qquad (2)$$

Second method: Weizsäcker formula

As a result of the nuclear interaction within the nucleus, each nucleon contributes positively to the binding energy of the nucleus, which is proportional to the number of nucleons (A) in the nucleus and the volume of the nucleus. The interactions of particles at the surface of the nucleus are similar to the surface tension phenomenon, which supports the spherical shape of the nucleus. Therefore, this term, known as the volume term, can be represented mathematically as [1]:

$$BE_{coul} = a_{vol}A \qquad (3)$$

Where a_{vol} is a constant whose value is greater than zero.

Nucleons near the surface of the nucleus are not surrounded by other nucleons, so they are less bound by the inward force of the nucleus. Therefore, it can be said that the number of surface nucleons is proportional to the area of the nuclear surface. Since this area of the quantum liquid droplet is spherical and proportional to the square of the nucleus radius r^2, and the nuclear radius is proportional to $A^{\frac{1}{3}}$, the surface term is equal to [10-11]:

$$BE_{sur} = 4\pi r^2 = 4\pi r_\circ^2 A^{\frac{2}{3}} = a_{sur}A^{\frac{2}{3}} \qquad (4)$$

The constant a_{sur} has a positive value more than zero.

The third term in this equation must take into account that all protons have a positive charge, and therefore experience repulsive Coulomb interactions with each other. Since the Coulomb potential is proportional to the square of the charge and inversely proportional to the radius of the nucleus, the Coulomb term is given by [5]:

$$BE_{coul} = \frac{3}{5}k\frac{(Ze)^2}{r} = a_{coul}Z^2A^{-\frac{1}{3}} \qquad (5)$$

The constant a_{coul} has a value greater than zero. This term distorts the nucleus and thus reduces the total binding energy of the nucleus.Also, due to the symmetry property of nucleons, we find that the total binding energy of the nucleus tends to decrease with increasing $(N - Z)$, and nuclei with $(N = Z)$ are more stable and have higher binding energy. This effect is known as the symmetry term, which contributes to the decrease in the binding energy of the nucleus and can be represented mathematically by the following relationship [12]:

$$BE_{symm} = a_{symm}(Z - N)^2A^{-1} \qquad (6)$$

As for the problem of the interaction of the two particles between pairs of nucleons, it depends on their relative spins. This in turn leads to high binding energies for nuclei in which all protons have double spin, and all neutrons also have double spin. Therefore, the even number of protons in the nucleus contributes positively to the binding energy equation, and in the same way, the even number of neutrons contributes positively. The pairing term in this theoretical model is defined as follows formula [3]:

$$BE_{pair} = \delta a_{pair}A^{-1/2} \qquad (7)$$

Table 1 shows the values of (δ) that affect the pairing term according to the values of (Z, N) in Equation (7) [13].

Table 1. Shows the values of (δ) that affect the pairing term according to the values of (Z, N) [13].

$\boldsymbol{\delta}$	**Z**	**N**
+1	Even	Even
0	Even	Odd

0	Odd	Even
-1	Odd	Odd

The sum of these binding energy terms is what is known as the Weizsäcker formula for the semi-empirical mass, given by the following form [3]:

$$BE(A,Z) = BE_{vol} + BE_{sur} + BE_{coul} + BE_{symm} + BE_{pair} \qquad (8)$$

$$BE(A,Z) = a_{vol}A - a_{sur}A^{\frac{2}{3}} - a_{coul}Z^2A^{-1/3} - a_{symm}(Z-N)^2A^{-1} \pm \delta a_{pair}A^{-1/2} \qquad (9)$$

Table 2. Shows the values of the constants used in equation (9) [3].

a_{vol} (MeV)	a_{sur} (MeV)	a_{coul} (MeV)	a_{sym} (MeV)	a_{pair} (MeV)
15.75	17.8	0.711	23.7	11.2

In order to calculate the difference between the practical values and the theoretical results that were calculated theoretically through equations (2) and (9) for the values of the binding energy, we apply the deviation equation, which is given by the following relationship [14-15]:

$$Deviaton = \left|\frac{BE_{CAL} - BE_{EXP}}{BE_{EXP}}\right| \times 100\% \qquad (10)$$

RESULTS AND DISCUSSION

In this study, the nuclear binding energy per nucleon was calculated for several different nuclei using both the fundamental equation and the Weizsäcker formula. The results obtained were compared with the experimental values, as shown in Table (3) [7].

The results of the study showed that the nuclear binding energy per nucleon values obtained from Equation (9) are more appropriate than the nuclear binding energy per nucleon values calculated using Equation (2). This is due to Equation (9) relying on both the atomic and mass numbers, in addition to taking into account the volume, surface, Coulomb, symmetry, and duality correction terms, while the fundamental equation (2) relied on the atomic and mass numbers and the nucleus mass only, without taking into account the correction terms mentioned above. As a result of the modifications made by Weizsäcker formula to his equation, it was observed through the deviation equation (10) that the deviation values matched. We note that the deviation value in the Weizsäcker formula is lower than the experimental values in the primary equation, with the exception of the hydrogen isotope (^3_1H), which exhibited a large deviation from the experimental value, equal to (78.4%). This is due to several reasons, including: the small atomic number, which reduces the effect of the Coulomb term $\left(\approx Z^2/A^{\frac{1}{3}}\right)$; the symmetry correction term ($\approx (Z-N)^2/A$), which leads to a decrease in the binding energy value due to the asymmetry between protons and neutrons; and the pairing term being equal to zero due to the difference between protons and neutrons, which leads to compensation ($\delta = 0$), according to Table (1).

Table 3. Shows the studied nuclei, their masses, binding energies per nucleon, and the amount of deviation from the experimental values.

Nuclei	A	Z	M_N (amu) [16,17]	B.E/A (MeV)			Deviation %	
				Primary	Weizsäcker	EXP [7]	(Primary & EXP)	(Weizsäcker & EXP)
H	3	1	3.016049	2.656948	0.610509	2.827	6.02	78.40
Mg	26	12	25.981538	8.135721	8.356565	8.33388	2.38	0.27
S	34	16	33.967867	8.342925	8.577516	8.5835	2.80	0.07
V	50	23	49.947163	8.460628	8.69309	8.69588	2.71	0.03
Fe	56	26	55.934942	8.5529	8.759902	8.79032	2.70	0.35
Ni	61	28	60.93106	8.530295	8.747604	8.76502	2.68	0.20
Kr	80	36	79.916378	8.462887	8.724501	8.69293	2.65	0.36
Sn	116	50	115.901744	8.302765	8.51634	8.52314	2.59	0.08
Hg	204	80	203.973476	7.685167	7.838417	7.88555	2.54	0.60
U	238	92	238.050783	7.372545	7.580554	7.57013	2.61	0.14

Nuclei with an intermediate mass number are more stable than those on either side, and these have the highest binding energy per nucleon of approximately 8.8 MeV. To further clarify the issue, we converted the numerical data in Table 3 into a Figure 1 showing three main regions with increasing mass numbers, as follows:

1. Nuclear fusion region: This is the region where the fusion of light atomic nuclei occurs to form heavier atomic nuclei, such as hydrogen.
2. Nuclear stability region: The BE(MeV) values are constant, and this curve shows a peak near $A = 60$, specifically near iron. This peak is known as the iron nucleus peak, which represents the most stable element.
3. Nuclear fission region: Nuclei with high mass numbers are less stable and are located beyond the iron nucleus peak.

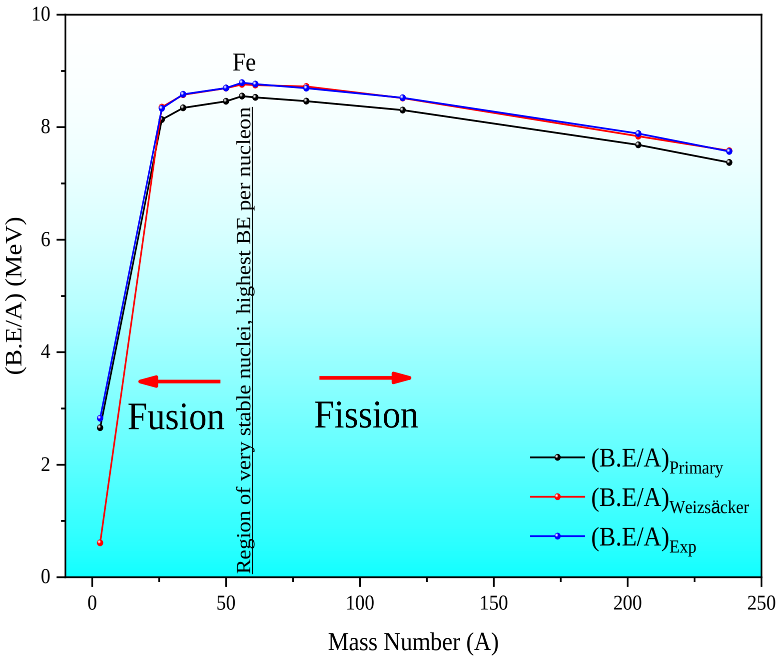

Figure 1. Shows the nuclear binding energy per nucleon as a function of mass number.

The results also showed that there is a great agreement between the practical values and the theoretically calculated values using equation (9) when the mass number increases, as it is clear from Table (3) that the percentage of deviation in the nucleus of the uranium atom with a mass number of 238 is equal to (0.14%), while in the basic equation the percentage of deviation was (2.61%) as following in Figure 2.

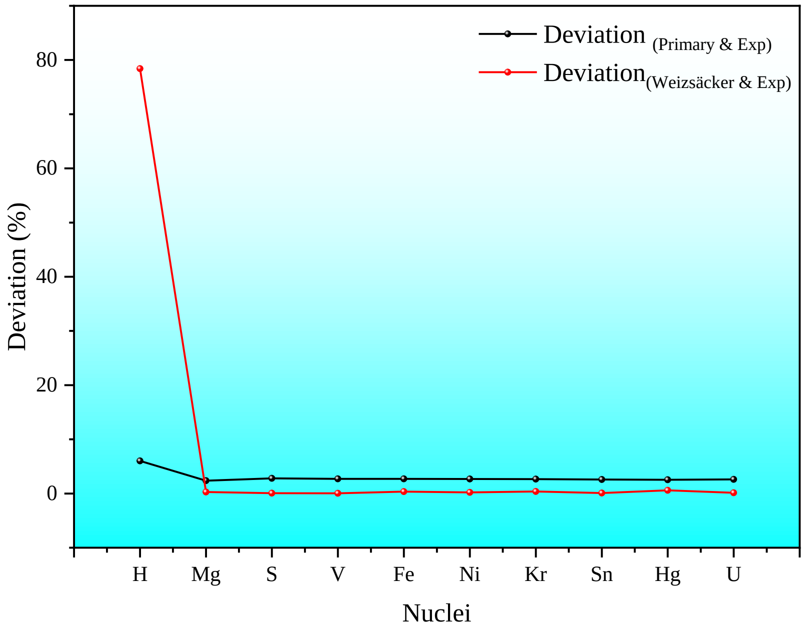

Figure 2. Shows the deviation between the theoretically calculated and the experimental values for the nuclear binding energy per nucleon.

CONCLUSION

Fundamental Finding : The results of the study showed that the nuclear binding energy per nucleon values obtained from Equation (9) are more appropriate than the nuclear binding energy per nucleon values calculated using Equation (2). This is due to Equation (9) relying on both the atomic and mass numbers, in addition to taking into account the volume, surface, Coulomb, symmetry, and pairing correction terms. **Implication :** The findings indicate that incorporating correction terms into the semi-empirical mass formula improves the accuracy of nuclear binding energy calculations, making the Weizsäcker formula more suitable for estimating binding energies across a wide range of nuclei. **Limitation :** The hydrogen isotope exhibited a large deviation from the experimental value, indicating that the Weizsäcker formula is less accurate for very light nuclei because the Coulomb, symmetry, and pairing terms contribute differently in such systems. **Future Research :** Future studies should evaluate additional theoretical models and apply them to a broader range of isotopes, particularly light nuclei, to improve the accuracy of nuclear binding energy predictions and reduce deviations from experimental data.

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